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# MOBILITY POWER FLOW ANALYSIS OF AN L-SHAPED PLATE STRUCTURE SUBJECTED TO DISTRIBUTED LOADING

J.M. Cuschieri and B. Cimmerman

Center For Acoustics and Vibration  
Department of Ocean Engineering  
Florida Atlantic University  
Boca Raton, Florida, USA.

## ABSTRACT

An analytical investigation based on the Mobility Power Flow Method is presented for the determination of the vibrational response and power flow for two coupled flat plate structures in an L-shaped configuration, subjected to distributed excitation. The principle of the Mobility Power Flow (MPF) method consists of dividing the global structure into a series of subsystems coupled together using mobility functions. Each separate subsystem is analyzed independently to determine the structural mobility functions for the junction and excitation locations. The mobility functions, together with the characteristics of the junction between the subsystems, are then used to determine the response of the global structure and the power flow. In the coupled plate structure considered here, mobility power flow expressions are derived for distributed mechanical excitation which is independent of the structure response. However using a similar approach with some modifications excitation by an acoustic plane wave can be considered. Some modifications are required to deal with the latter case are necessary because the forces (acoustic pressures) acting on the structure are dependent on the response of the structure due to the presence of the scattered pressure.

## 1. INTRODUCTION

Structural power flow methods, both experimental and analytical have been used for the analysis of the vibration response and structural power flow transmission of structural elements for a number of years. Noiseux [1] published one of the earliest works on the theory and application of the experimental measurement of structural intensity. Other work by Pavic [2], Verheij [3] and Redman-White [4], to name just a few, followed in developing methods and approaches to measure structural intensity. One of the most used text on the analytical description of structural intensity is the book by Cremer, Heckl and Ungar [5]. However, there is also a significant volume of work by others such as Skudrzyk [6], Dieter and Ungar [7], etc. who

either deal directly with structural power flow concepts or consider topics which are very closely related.

The analytical description of the flow of structural intensity through connected structures was formalized in the work by Goyder and White [8], Verheij [9] and later on by Pinnington and White [10], who developed expressions that described the structural intensity flow between connected components of a structure in terms of mobility functions of the connected structures. The work done in these references mainly addressed point connected structures and/or structures of infinite extent. Important contributions from these works are the demonstrated potential of the power flow techniques to describe the behavior and the interaction between connected structures and the potential of these techniques to track the flow of power which can lead to important information in the control of the response or radiated noise from a structure.

This approach, the use of structural mobility functions to describe the flow of structural intensity between coupled structures, can be used as an analysis tool to describe the dynamic response of complex structures. Referring to this approach as the Mobility Power Flow (MPF) method, it would be similar to such other analysis techniques such as Statistical Energy Analysis (SEA) or Finite Element Analysis (FEA). In the MPF approach the global structure is considered to be made up of a number of connected substructures or subsystems, which represent either physical partitions in the global structure or different wave components that can propagate through the structure. This is identical to the substructuring used in the application of SEA. The same advantages of SEA would therefore apply for the MPF method. That is, if modifications are made to any one of the subsystems, only the evaluation of the terms associated with that subsystem and the interaction analysis need to be repeated.

The difference between SEA and MPF is that, while in SEA the coupling between the substructures is described in terms of a coupling loss factors, which do not take into consideration the modal behavior of the substructures, in the MPF method the coupling is described by the mobility functions which can be made to represent the modal behavior. At high frequencies the two techniques would converge onto the same result, except that the SEA method is much more efficient computationally. However, at medium to low frequencies, because of the modal response of the structure, the behavior of the structure can significantly deviate from the mean average response, as obtained using an SEA approach. The MPF approach retains a description of the modal behavior of the structure and therefore can give better results, closer to the actual behavior of the structure. The MPF method can be more appropriate in these medium frequencies, provided the structure can be accurately modelled.

Compared to FEA, the MPF method has the advantage of the substructuring which is significant for large complex structures. The results obtained by FEA and MPF are identical. However, the MPF method is not an exclusive tool for the determination of the power flow or response of a complex structure, but a complementary tool that can be used together with these other techniques. The results obtained by the MPF can be matched to results obtained using FEA at low frequencies and SEA at high frequencies [11].

The MPF technique describes the structural power flow between the subsystems in terms of the structural mobility functions at the excitation locations and at the junctions between the subsystems. The implementation of this technique requires prior knowledge of these mobility functions. Depending on the type of structure and the number of subsystems used in describing the global structure, the number of mobility functions to be determined can be significant. In the case of periodic or quasi-periodic structures, the mobility functions of each of the subsystems will in general be identical and the application of the MPF method is particularly efficient for these

types of structures [12]. In reference [12] the application of the MPF method to the analysis of a periodic beam with a finite number of spans is demonstrated.

## 2. POWER FLOW IN AN L-SHAPED PLATE

In the introduction a general discussion on the MPF method was presented, where most of the cited work dealt with either point connected substructures or alternatively, if line connections are considered the structures are considered infinite in extent. In this section the development of the expressions for the power flow between two finite plates joined along a common edge to form an L-shape are presented. The loading on one of the plates is considered to be a distributed mechanical load, where the load is independent of the response of the plates. The case of point loading is a special case of distributed loading where the distribution function can be mathematically represented by a Dirac function. Since results have been obtained for an L-shaped plate with mechanical point loading in [13], the results from this analysis for the distributed load will be compared to the results obtained for the point loading before other examples of distributed loading will be considered to verify the analysis.

In deriving the power flow expressions the following assumptions are made:

- (a) The plates are thin compared to the minimum wavelength. That is shear deformation and rotary inertia are neglected.
- (b) Only one type of waves, bending waves are considered. (See comments in the conclusion regarding other wave types).
- (c) The junction between the two plates is pinned. Since the two plates are connected at right angles, and in-plane waves are not considered, this assumption greatly simplifies the analysis.
- (d) The junction between the two plates is rigid, that is the angle between the plates is always 90 degrees.
- (e) The remaining edges of the two plates are simply supported, that is with zero bending moment and deflection.

In the case of the L-shaped plate (figure 1) with only bending waves, the MPF model consists of two plate substructures. Coupling between the two substructures is defined in terms of the mobility functions of each of the two substructures considered separately. For two coupled general subsystems (figure 2) the input and transfer power flow expressions are given by [10]:

$$\text{Power}_{\text{input}} = \frac{1}{2} |F(f)|^2 \text{Real} \left[ M_1 - \frac{M_{12}M_{21}}{M_2 + M_3} \right] \quad (1)$$

$$\text{Power}_{\text{trans}} = \frac{1}{2} |F(f)|^2 \left| \frac{M_{21}}{M_2 + M_3} \right|^2 \text{Real} \left[ M_3 \right] \quad (2)$$

where  $M_i$ ;  $i=1,2$  and  $3$ , are the input mobilities at locations  $1$ ,  $2$  and  $3$  respectively,  $M_{12}$  and  $M_{21}$  are the transfer mobilities between

locations 1 and 2 and  $|F(f)|^2$  is the power spectrum of the excitation load.

Extending these equations to the case of the two plates joined along a common edge to form an L-shape, with a distributed load on one of the plates, the loaded plate will be referred to as the source plate and the other plate is the receiver plate, the general form of the expressions will not change but the definition of the mobility function changes since these become functions of the spatial coordinates  $x$  and  $y$ . Also, because of the load distribution on the surface of the source plate, and the line junction between the two plates, integrals over the surface and the length of the junction have to be taken to determine respectively the input and transferred power.

The integrals in the power flow expressions, can complicate the use of the MPF technique for any general structure. However, a way around these integrals is to use a spatial or modal decomposition. In this way, the same expressions for the power flow as in equations (1) and (2) can be used, with the exception that the mobility functions in these expressions would represent mobility functions either in the wavespace domain - the spatial transform would transform the spatial coordinates into wavespace coordinates - or the modal domain. For the L-shaped plate considered here, with the edges simply supported, a modal decomposition can be used and the two equations (1) and (2) can be rewritten in the form:

$$\text{Power}_{\text{input}} = \frac{1}{2} \text{Real} \left\{ \sum_{m,n} \frac{ab}{4} |F_{mn}|^2 M_{1mn} - \frac{a}{2} \sum_m \frac{M_{21m}}{M_{2m} + M_{3m}} |F_m|^2 \int_0^b f(y)^* M_{12m}(y) dy \right\} \quad (3)$$

$$\text{Power}_{\text{trans}} = \frac{1}{2} \frac{a}{2} \sum_m \left| \frac{M_{21m} F_m}{M_{2m} + M_{3m}} \right|^2 \text{Real} [M_{3m}] \quad (4)$$

where  $m$  and  $n$  are respectively the mode number for the  $x$  and  $y$  directions, that is  $m$  is along the junction and  $n$  perpendicular to the junction, and  $a$  and  $b$  are the dimensions of the plates (figure 1). If  $F(x,y)$  is the surface distributed load, then in equations (3) and (4) the following definitions apply;

$$F(x,y) = F_0 f(x)f(y) \quad (5)$$

$$F_m = \frac{2}{a} \int_0^a F_0 f(x) \sin \left( \frac{m\pi x}{a} \right) dx \quad (6)$$

$$F_n = \frac{2}{b} \int_0^b f(y) \sin\left[\frac{n\pi y}{b}\right] dy \quad (7)$$

$$F_{mn} = F_m F_n \quad (8)$$

The mobility functions in equations (3) and (4) are defined according to the assumptions made for the L-shaped structure. A generic approach can be used but this will make the analysis unnecessarily complicated. Therefore,  $M_{2m}$  and  $M_{3m}$  are input modal mobility functions defined by the ratio of the mode  $m$  component of the rate of change of slope (angular velocity) at an edge of the plate, to a line moment applied along the same edge. The moment distribution along the plate edge is in the form of the eigenfunction for mode  $m$ .  $M_{1mn}$  is the input modal mobility defined as the ratio of mode  $(m,n)$  component of the transverse velocity on the surface of the plate, to an applied distributed surface load with the same distribution as the eigenfunction for mode  $m,n$ .  $M_{21m}$  is a transfer mobility defined as the ratio of the mode  $m$  component of a plate edge angular velocity, to the mode  $m$  component of an applied distributed load on the surface of the plate.  $M_{12m}(y)$  is also a transfer mobility defined by the ratio of the mode  $m$  component of plate surface transverse velocity at any point  $y$ , to the mode  $m$  component of an edge moment.

As can be observed from these definitions of the mobility functions, these do not consider the global structure (L-shaped plate). Each mobility function is defined with reference to a flat plate substructure. Some restrictions are imposed on which mobility functions are used in the analysis, but this is solely for the purpose of simplifying the problem for this presentation. A more general model can be used if so desired [12].

The analysis of the global L-shaped structure is therefore reduced to that of the substructures. The total behavior - power flow - of the global structure is determined through the use of equations (3) and (4).

### 3. MOBILITY EXPRESSIONS

The evaluation of the mobility functions  $M_{1mn}$  and  $M_{21m}$  can be achieved by considering a plate structure with a distributed load (figure 3). Solving for plate equation of motion,

$$M_{1mn} = \frac{\dot{u}_{mn}}{F_{mn}} = \frac{jf}{2\pi\rho h} \frac{1}{\left[f_{mn}^*\right]^2 - f^2} \quad (9)$$

and

$$M_{21m} = \frac{\dot{\theta}_m}{F_m} = \frac{jf}{2\rho hb} \sum_n \frac{(-1)^n n F_n}{\left[f_{mn}^*\right]^2 - f^2} \quad (10)$$

where

$$f_{mn}^* = \frac{1}{2\pi} \sqrt{\frac{D^*}{\rho h} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2} \right]} \quad (11)$$

and  $u_{mn}$  is the mode  $m, n$  component of the plate surface displacement,  $\theta_m$  is the mode  $m$  component of the angular displacement at the edge of the plate and  $D^*$  and  $\rho h$  are the plate flexural rigidity and surface density respectively.

To evaluate the mobility functions  $M_{2m}$ ,  $M_{3m}$  and  $M_{12m}$  a plate structure with a distributed edge moment is considered (figure 4). In this case the following mobility functions are obtained [13],

$$M_{2m} = M_{3m} = \frac{\dot{\theta}_m}{T_m} = \frac{j}{2\sqrt{\rho h D^*}} \left[ \frac{k_2}{\tan(k_2 b)} - \frac{k_1}{\tanh(k_1 b)} \right] \quad (12)$$

$$M_{12m}(y) = \frac{\dot{u}_m(y)}{T_m} = \frac{j}{2\sqrt{\rho h D^*}} \left[ \frac{\sin(k_2 y)}{\sin(k_2 b)} - \frac{\sinh(k_1 y)}{\sinh(k_1 b)} \right] \quad (13)$$

where  $T_m$  is the mode  $m$  component of the edge moment and  $k_1$  and  $k_2$  are defined by:

$$\begin{aligned} k_1^2 &= 2k_x^2 + k_y^2 ; & k_2 &= k_y \\ k_x &= m\pi/a ; & k_y^2 &= \omega \sqrt{\frac{\rho h}{D^*}} - k_x^2 \end{aligned} \quad (14)$$

#### 4. RESULTS

The results that will be presented for the power flow consider the L-shaped plate with the characteristics shown in figure (1). The two plates forming the L-shape are identical. This condition is selected to simplify the analysis and is not a restriction on the application of the MPF method. Two loading conditions are considered; (a) a point load and (b) a uniformly distributed load. The power flow results, input and transferred are normalized with respect to the total applied load.

##### 4.1 Point Load

The force is in this case described mathematically by;

$$F(x, y) = F_0 \delta(x - x_0) \delta(y - y_0) \quad (15)$$

and

$$F_{mn} = \frac{4}{ab} F_o \sin \left[ \frac{m\pi x_o}{a} \right] \sin \left[ \frac{n\pi y_o}{b} \right] \quad (16)$$

where  $x_o$  and  $y_o$  are the  $x$  and  $y$  coordinates of ~~the point~~ of application of the load. If the load is applied at the center of the plate ( $x_o = a/2$  and  $y_o = b/2$ ), then only odd values for  $m$  and  $n$  are allowed. Substituting in equations (3) and (4), the results obtained for the input and transferred power, when the load is in the center of the source plate, are shown in figure (5). Comparing these results to those obtained in reference [13], the two sets of results are identical. This verifies the formulation of the power flow expressions in the form shown in equations (3) and (4).

#### 4.3 Uniform Distributed Load

The force applied on the source plate is described by:

$$F(x,y) = F_o \quad (17)$$

$$f(y) = 1.0 \quad (18)$$

and

$$F_{mn} = \frac{16 F_o}{\pi^2 (2m+1)(2n+1)} \quad (19)$$

The solution follows in the same way as for the point load case. The results for the power flow are shown in figure (6).

Comparing the results for the power flow obtained for the point load with the results obtained for the distributed load, the following similarities and differences can be observed. First, the modes excited by the two types of loading are identical. This is expected since both loading conditions are symmetrical about the center of the source plate. Second, the general level of the power flow for the distributed load case decreases with frequency, while that for the point load does not decrease. The reason for this can be mathematically described by the inverse dependency of the modal components of the distributed applied load on the mode number for the case of the distributed load (equation (19)). Physically this implies that the higher frequency modal components of the load are suppressed. The results for the distributed load would be similar to those for excitation by normal incidence acoustic plane waves, if the scattered pressure component is neglected.

#### 5. CONCLUSION

The mobility power flow approach for the analysis of coupled plate structures with distributed load excitation has been described in this paper. Some assumptions have been made regarding the MPF model for the L-shaped plate. With regards to the selected boundary conditions, these can be relaxed to deal with other boundaries but the analysis can get more complicated. If other boundary conditions are selected for which a modal decomposition is possible, then the same type of analysis can be performed.

If in-plane waves were to be considered, the MPF method can be used, but in this case each wave component would be described by a separate subsystem. That is, the L-shaped plate would be modelled by six subsystems, three for each plate, representing respectively

bending waves, in-plane longitudinal waves and in-plane shear waves. The power flow expressions would in this case include more mobility functions and it would be more appropriate to use a matrix representation [12] for the solution to the problem. Because no in-plane waves are considered here, the junction between the two plates is not allowed to have lateral movement. Only rotational motion is allowed. Therefore, the assumption of a pinned condition at the junction already includes the fact that no translational motion is allowed at the junction, which reduces the size of the MPF model, requiring a lower number of mobility functions.

The overall conclusion that can be made is that power flow techniques can be used as an analysis tool similar to other method of structural analysis. The need to evaluate the mobility functions can make the approach difficult to implement on certain types of structures, but for beam, plate and periodic structures, the MPF method can represent a very powerful tool for the estimation of the modal behavior of the structure. The MPF method evaluates the power flow through the coupled structure. The understanding of how the vibrational power is propagated through a build up structure can in certain instances, be more of interest than just the vibrational response at some point on the structure.

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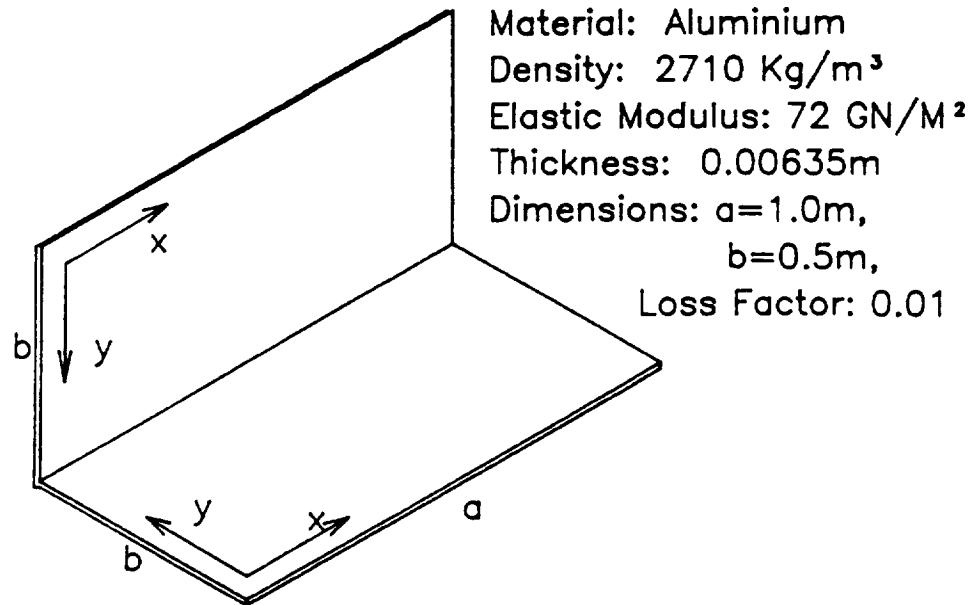


Figure 1. L-shaped plate structure showing plate characteristics.

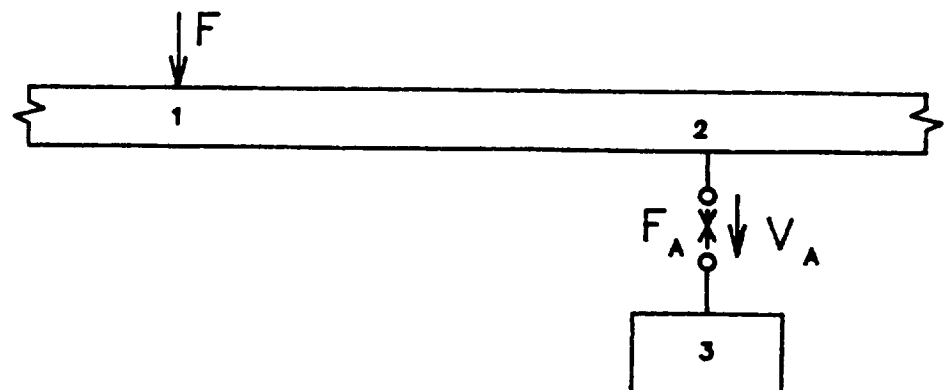


Figure 2. General mobility power flow model.

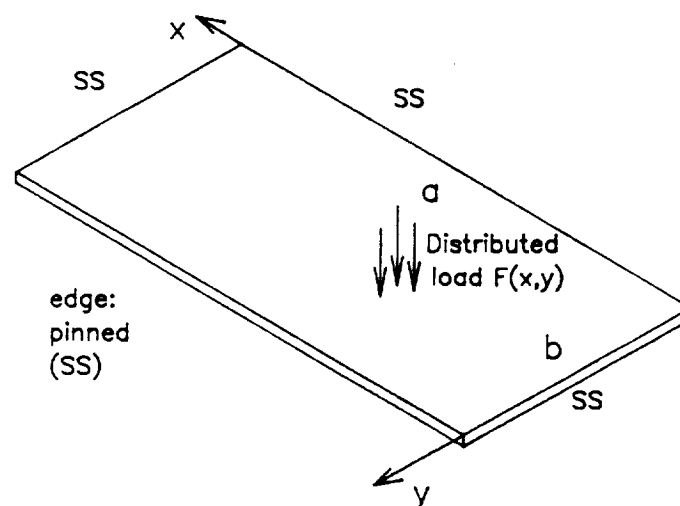


Figure 3. Plate with distributed surface load.

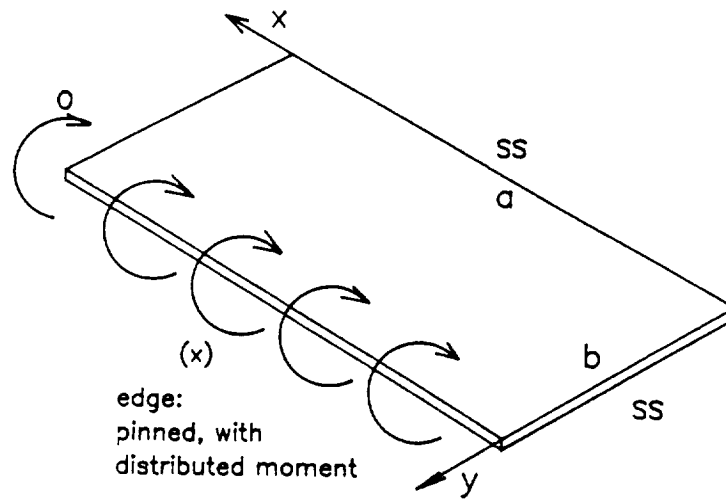


Figure 4. Plate with distributed edge moment.

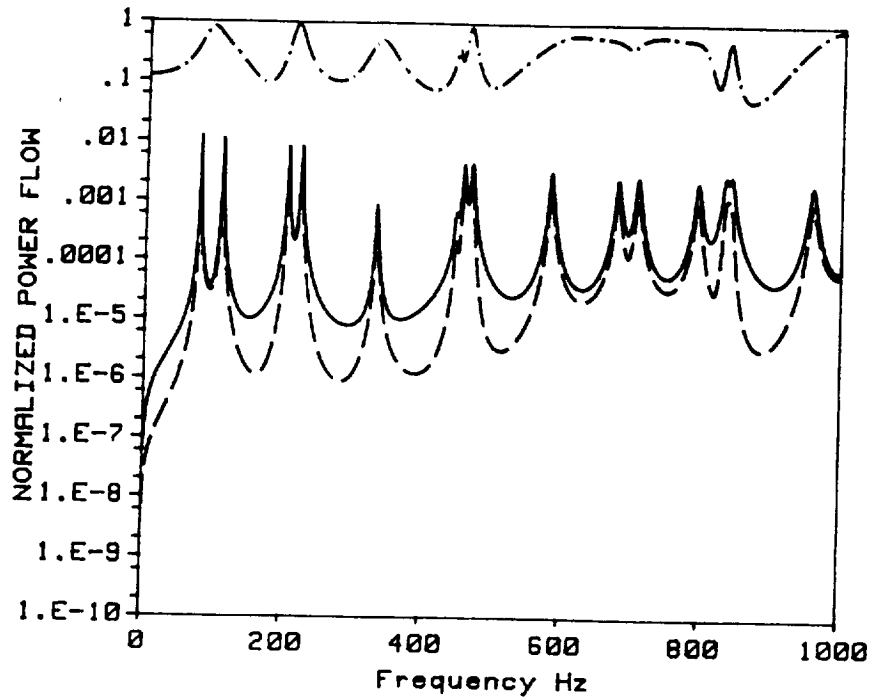


Figure 5. Normalized power flow results for point loading. —: Power input; ----: power transfer; -.-.: power ratio.

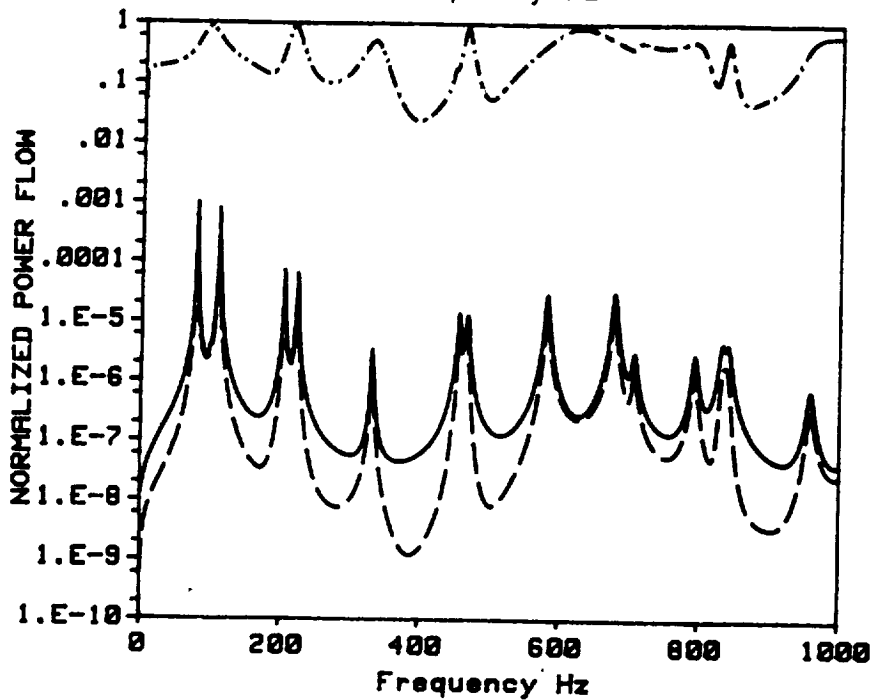


Figure 6. Normalized power flow results for distributed loading. —: Power input; ----: power transfer; -.-.: power ratio.